**Kaden Fuller-Aujla**

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Chapter 1:

# **Definition 1.1 - Mean**

The *mean* of a sample of n measured responses … is given by

The corresponding population mean is denoted mu

# **Definition 1.2 - Variance**

The *variance* of a sample of measurements y1, y2,..., yn is the sum of the square of the differences between the measurements and their mean, divided by n − 1. Symbolically, the sample variance is

The corresponding population variance is denoted by the symbol σ2.

# **Definition 1.3 – Standard Deviation**

The standard deviation of a sample of measurements is the positive square root of the variance; that is,

The corresponding population standard deviation is denoted by

**Chapter 2:**

# **Definition 2.1 - Experiment**

An *experiment* is the process by which an observation is made.

# **Definition 2.2 - Simple Event**

A *simple event* is an event that cannot be decomposed. Each simple event corresponds to one and only one *sample point.* The letter *E* with a subscript will be used to denote a simple event or the corresponding sample point.

# **Definition 2.3 - Sample Space**

The *sample space* associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by *S.*

# **Definition 2.4 - Discrete Sample Space**

A *discrete sample* space is one that contains either a finite or a countable number of distinct sample points.

# **Definition 2.5 - Event**

An *event* in a discrete sample space *S* is a collection of sample points—that is, any subset of *S*.

# **Definition 2.6 - Axioms that form definition of Probability**

Suppose *S* is a sample space associated with an experiment. To every event *A* in *S* (*A* is a subset of *S*), we assign a number, *P(A),* called the probability of *A*, so that the following axioms hold:

Axiom 1: *P(A) ≥* 0.

Axiom 2: *P(S)* = 1.

Axiom 3: If … form a sequence of pairwise mutually exclusive events in

*S* (that is, then

# **Definition 2.7 - Permutation**

An ordered arrangement of *r* distinct objects is called a *permutation.* The number of ways of ordering *n* distinct objects taken *r* at a time will be designated by the symbol .

# **Definition 2.8 - Combination**

The number of *combinations* of *n* objects taken *r* at a time is the number of subsets, each of size *r,* that can be formed from the *n* objects. This number will be denoted by .

# **Definition 2.9 - Conditional Probability**

The *conditional probability of an event A*, given that an event *B* has occurred, is equal to

provided P(B) > 0. [The symbol P(A|B) is read “probability of A given B.”]

# **Definition 2.10 - Independence**

Two events *A* and *B* are said to be *independent* if any one of the following holds:

*P(A|B) = P(A),*

*P(B|A) = P(B),*

Otherwise, the events are said to be *dependent.*

# **Theorem 2.5 - The Multiplicative Law of Probability**

The probability of the intersection of two events A and B is

If A and B are independent, then

The multiplicative law follows directly from Definition 2.9, the definition of conditional probability.

# **Theorem 2.6 - The Additive Law of Probability**

The probability of the union of two events A and B is

If A and B are mutually exclusive events, and

# **Theorem 2.7 - Relationship between**

If A is an event, then

# **Definition 2.11 - Partition**

For some positive integer *k,* let the sets be such that

Then the collection of sets is said to be a *partition* of S.

# **Theorem 2.9 - Bayes’ Rule**

Assume that {} is a partition of S (see definition 2.11) such that

Then

# **Definition 2.12 - Random Variable**

A *random variable* is a real-valued function for which the domain is a sample space.

# **Definition 2.13 - Random Sample**

Let *N* and *n* represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the ) samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a *random sample.*

Chapter 3:

# **Definition 3.1 - Discrete**

A random variable Y is said to be discrete if it can assume only a finite or countably number of distinct values.

# **Definition 3.2 - Sum of all probabilities of all sample points in S**

The probability that *Y* takes on the value *y,* *P(Y = y),* is defined as the *sum of the probabilities of all sample points in S* that are assigned the value y. We will sometimes denote *P(Y = y)* by *p(y).*

# **Definition 3.3 - Probability Distribution**

The *probability distribution* for a discrete variable *Y* can be represented by a formula, a table, or a graph that provides *p(y) = P(Y = y)* for all *y.*

# **Definition 3.4 - Expected Value**

Let Y be a discrete random variable with the probability function p(y). Then the expected value of Y, E(Y ), is defined to .

# **Definition 3.5 - Standard Deviation of distribution given by p(y)**

If Y is a random variable with mean E(Y ) = *µ,* the variance of a random variable *Y* is defined to be the expected value of *.* That is,

The *standard deviation* of *Y* is the positive square root of *V(Y).*

# **Definition 3.6 - Binomial Experiment**

A *binomial experiment* possesses the following properties:

1. The experiment consists of a fixed number, n, of identical trials.

2. Each trial results in one of two outcomes: success, S, or failure, F.

3. The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to q = (1 − p).

4. The trials are independent.

5. The random variable of interest is Y , the number of successes observed during the n trials.

# **Definition 3.7 - Binomial Distribution**

A random variable *Y* is said to have a *binomial distribution* based on *n* trials with success probability *p* if and only if

# **Definition 3.8 - Geometric Probability Distribution**

A random variable Y is said to have a *geometric probability distribution* if and only if

# **Theorem 3.8 - Geometric Distribution**

If *Y* is a random variable with a geometric distribution,

# **Definition 3.9 - Negative Binomial Probability Distribution**

A random variable Y is said to have a *negative binomial probability distribution* if and only if

# **Theorem 3.9- Random Variable Negative Binomial Distribution**

If Y is a random variable with a negative binomial distribution,

# **Definition 3.10- Hypergeometric Probability Distribution**

A random variable Y is said to have a *hypergeometric probability distribution* if and only if

where y is an integer 0, 1, 2,..., n, subject to the restrictions y ≤ r and n − y ≤ N − r.

# **Theorem 3.10- Random Variable Hypergeometric Distribution**

If Y is a random variable with a hypergeometric distribution,

# **Definition 3.11- Poisson Probability Distribution**

A random variable Y is said to have a *Poisson probability distribution* if and only if

# **Theorem 3.11- Random Variable Poisson Distribution**

If Y is a random variable possessing a Poisson distribution with parameter λ, then

By definition,

Notice that the first term in this sum is equal to 0 (when y = 0), and, hence,

# **Definition 3.12- Kth Moment of a Random Variable Y Taken about the Origin**

The *kth moment of a random variable Y taken about the origin* is defined to be E(Y k ) and is denoted by µk .

# **Definition 3.13- Kth Moment of a Random Variable Taken about its Mean, Kth central Moment of Y**

*The kth moment of a random variable Y taken about its mean*, or the *kth central moment of Y* , is defined to be E[(Y − µ)k ] and is denoted by µk .

# **Definition 3.14- Moment-Generating function m(t) for Y**

The *moment-generating function m(t) for a random variable Y* is defined to be m(t) = E(etY ). We say that a moment-generating function for Y exists if there exists a positive constant b such that m(t) is finite for |t| ≤ b.

# **Theorem 3.12- If m(t) exists for K**

f m(t) exists, then for any positive integer k,

In other words, if you find the kth derivative of m(t) with respect to t and then set t = 0, the result will be µ k .

# **Definition 3.15- The Probability-Generating Function P(t)**

Let Y be an integer-valued random variable for which P(Y = i) = pi , where i = 0, 1, 2,... . The *probability-generating function P(t)* for Y is defined to be

for all values of t such that P(t) is finite.

# **Definition 3.16- Kth Factorial Moment**

The *kth factorial moment* for a random variable Y is defined to be

where k is a positive integer.

# **Theorem 3.13- Probability Generating Function for Random Variable**

If P(t) is the probability-generating function for an integer-valued random variable, Y , then the kth factorial moment of Y is given by

# **Theorem 3.14- Tchebysheff’s Theorem**

**Tchebysheff’s Theorem** Let Y be a random variable with mean µ and finite variance σ2. Then, for any constant k > 0,

Chapter 4:

# **Definition 4.1- Distribution Function of Y**

Let Y denote any random variable. The distribution function of Y , denoted by F(y), is such that F(y) = P(Y ≤ y) for < y < .

# **Theorem 4.1- Properties of Distribution Function**

Properties of a Distribution Function1 If F(y)is a distribution function, then

1. F() ≡ lim y→F(y) = 0.

2. F() ≡ lim y→F(y) = 1.

3. F(y) is a nondecreasing function of y. [If y1 and y2 are any values such that y1 < y2, then F(y1) ≤ F(y2).]

# **Definition 4.2- Continuous Distribution Function**

A random variable Y with distribution function F(y) is said to be continuous if F(y) is continuous, for < y <

# **Definition 4.3- Probability Density Function**

Let F(y) be the distribution function for a continuous random variable Y . Then f (y), given by

Wherever the derivative exists, is called the *probability density function* for the random variable Y .

# **Theorem 4.2- Properties of a Density Function**

**Properties of a Density Function** If f (y)is a density function for a continuous random variable, then

# **Definition 4.4- Random Variable for Percentile**

Let Y denote any random variable. If 0 < p < 1, the pth quantile of Y , denoted by φp, is the smallest value such that P(Y ≤ φq ) = F(φp) ≥ p. If Y is continuous, φp is the smallest value such that F(φp) = P(Y ≤ φp) = p. Some prefer to call φp the 100pth percentile of Y .

# **Theorem 4.3- Density Function Falling in the Interval**

If the random variable Y has density function f (y) and a < b, then the probability that Y falls in the interval [a, b] is

# **Definition 4.5- Expected Value of a Continuous Random Variable Y**

The expected value of a continuous random variable Y is

provided that the integral exists.

# **Theorem 4.4- Expected Value of g(Y) Provided the Integral Exists**

Let g(Y ) be a function of Y ; then the expected value of g(Y ) is given by

provided that the integral exists.

# **Theorem 4.5- C, constant and g(Y) be functions of Continuous Random Variable Y**

Let c be a constant and let g(Y ), g1(Y ), g2(Y ), . . . , gk (Y ) be functions of a continuous random variable Y . Then the following results hold:

1. E(c) = c.

2. E[cg(Y )] = cE[g(Y )].

3. E[g1(Y )+g2(Y )+· · ·+gk (Y )] = E[g1(Y )]+E[g2(Y )]+· · ·+E[gk (Y )].

# **Definition 4.6- Uniform Probability Distribution**

If , a random variable Y is said to have a *continuous uniform probability distribution* on the interval () if and only if the density function of Y is

# **Definition 4.7- Parameters of the Density Function**

The constants that determine the specific form of a density function are called *parameters* of the density function.

# **Theorem 4.6- Random Variable Uniformly Distributed**

If and Y is a random variable uniformly distributed on the interval (), then

# **Definition 4.8- Normal Probability Distribution**

A random variable Y is said to have a normal probability distribution if and only if, for σ > 0 and <µ< , the density function of Y is

# **Theorem 4.7- Random Variable Normally Distributed**

If Y is a normally distributed random variable with parameters µ and σ, then

# **Definition 4.9- Gamma Distribution with Parameters**

A random variable Y is said to have a *gamma distribution with parameters* α > 0 and β > 0 if and only if the density function of Y is

Where

# **Theorem 4.8- Gamma Distribution with Parameters α and β**

If Y has a gamma distribution with parameters α and β, then

# **Definition 4.10**

Let ν be a positive integer. A random variable Y is said to have a *chi-square distribution with ν degrees of freedom* if and only if Y is a gamma-distributed random variable with parameters α = ν/2 and β = 2.

# **Theorem 4.9- T Is Chi-Squared Random Variable**

If Y is a chi-square random variable with ν degrees of freedom, then

# **Definition 4.11- Exponential Distribution with Parameter**

A random variable Y is said to have an *exponential distribution with parameter* β > 0 if and only if the density function of Y is

# **Theorem 4.10- If Y is an Exponential Random Variable**

If Y is an exponential random variable with parameter β, then

# **Definition 4.12- Beta Probability Distribution with Parameters**

A random variable Y is said to have a *beta probability distribution with parameters* α > 0 and β > 0 if and only if the density function of Y is

Where

# **Theorem 4.11- Beta Distributed with Random Variable**

If Y is a beta-distributed random variable with parameters α > 0 and β > 0, then

# **Definition 4.13- If Y is cont. and random, Kth Moment About the Origin**

If Y is a continuous random variable, then the *kth moment about the origin* is given by

The *kth moment about the mean*, or the kth central moment, is given by

# **Definition 4.14- Moment- Generating Function of Y**

If Y is a continuous random variable, then the moment-generating function of Y is given by

The moment-generating function is said to exist if there exists a constant b > 0 such that m(t) is finite for |t| ≤ b.

# **Theorem 4.12- Random Variable with Density Function, with Moment Generating Function**

Let Y be a random variable with density function f (y) and g(Y ) be a function of Y . Then the moment-generating function for g(Y ) is

# **Theorem 4.13- Tchebysheff’s Theorem**

**Tchebysheff’s Theorem** Let Y be a random variable with finite mean µ and variance σ2. Then, for any k > 0,

# **Definition 4.15- Mixed Distribution Functions**

Let Y have the mixed distribution function

and suppose that X1 is a discrete random variable with distribution function F1(y) and that X2 is a continuous random variable with distribution function F2(y). Let g(Y ) denote a function of Y . Then

Chapter 5:

# **Definition 5.1- Joint Probability Function**

Let be discrete random variables. The joint (or bivariate) probability function for is given by

# **Theorem 5.1- Discrete Random Variables with Joint Probability Function**

If are discrete random variables with joint probability function p(), then

# **Definition 5.2- Random Variables with Joint Distribution Function**

For any random variables, the joint (bivariate) distribution function F() is

# **Definition 5.3- Continuous Random Variables with Joint Distribution Function**

Let be continuous random variables with joint distribution function F(). If there exists a nonnegative function f (), such that

for all are said to be jointly continuous random variables. The function f () is called the joint probability density function

# **Theorem 5.2**

If are random variables with joint distribution function F(), then

1. F() = F() = F( ) = 0.

2. F() = 1.

3. If y∗ 1 ≥ y1 and y∗ 2 ≥ y2, then F(y∗ 1 , y∗ 2 ) − F(y∗ 1 , y2) − F(y1, y∗ 2 ) + F(y1, y2) ≥ 0.

# **Definition 5.4- Marginal Probability Functions**

# **Definition 5.5- Conditional Discrete Probability Function of Y**

If are jointly discrete random variables with joint probability function p(y1, y2) and marginal probability functions and p2(y2), respectively, then the conditional discrete probability function of is

provided that

# **Definition 5.6- Conditional Distribution Function of Y**

If are jointly continuous random variables with joint density function f (), then the conditional distribution function of given is

# **Definition 5.7- Jointly Continuous Random Variables with Joint Density**

Let be jointly continuous random variables with joint density f () and marginal densities is given by

and, for any is given by

# **Definition 5.8**

Let have distribution function have distribution function

for every pair of real numbers are not independent, they are said to be dependent.

# **Theorem 5.4**

If are discrete random variables with joint probability function p() and marginal probability functions , respectively, then are independent if and only if

for all pairs of real numbers ().

# **Theorem 5.5**

Let have a joint density f () that is positive if and only if a ≤ ≤ b and c ≤ ≤ d, for constants a, b, c, and d; and f (,) = 0 otherwise. Then are independent random variables if and only if

where is a nonnegative function of alone and h() is a nonnegative function of alone.